Math 53: Multivariable Calculus | **Sections 201, 204**

Worksheet for 2021-09-03

Conceptual questions

Question 1. If $\mathbf{u}, \mathbf{v}$ are vectors of lengths 2 and 3 respectively, what are the largest and smallest possible values of $\mathbf{u} \cdot \mathbf{v}$? Draw pictures of both situations.

Question 2. If $\mathbf{r} = \langle x, y \rangle$, $\mathbf{a} = \langle a_1, a_2 \rangle$, and $\mathbf{b} = \langle b_1, b_2 \rangle$, expand out the equation

$$(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$$

and say what kind of shape it is. Can you interpret the vector equation of this shape geometrically?

Question 3. The following are true for vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$:

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta$$

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \theta$$

where θ is the angle between them. Given $\mathbf{u}, \mathbf{v}$ and asked for θ, which of the above equations would you use, and why?

Computations

Problem 1. Consider the cube with opposite corners $(0, 0, 0)$ and $(2, 2, 2)$, whose edges are parallel to the coordinate axes in $\mathbb{R}^3$. The intersection of this cube with the plane $x + y + z = 3$ is a hexagon! Show that this hexagon is *regular*, meaning that all of its edges are the same length, and that all of its interior angles are the same as well.